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Supersymmetic Extension of Non-Abelian Scalar-Tensor Duality

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Abstract

The field theory dual to the Freedman-Townsend model of a non-Abelian anti-symmetric tensor field is a nonlinear sigma model on the group manifold G. This can be extended to the duality between the Freedman-Townsend model coupled to Yang-Mills fields and a nonlinear sigma model on a coset space G/H. We present the supersymmetric extension of this duality, and find that the target space of this nonlinear sigma model is a complex coset space, $G^{\mathbf{C}}/H^{\mathbf{C}}$.

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1 Introduction

The electric-magnetic duality plays a crucial role in studies of non-perturbative dynamics of four-dimensional supersymmetric gauge theories [1]. In four dimensions, there exists another duality between scalar fields and anti-symmetric tensor (AST) fields, called the scalar-tensor duality [2]. AST fields $B(x) = \frac{1}{2}B_{\mu\nu}(x)dx^{\mu} \wedge dx^{\nu}$ are an important ingredient in supergravity and superstring [3, 4]. The massless theory of the AST field possesses AST gauge invariance: $B(x) \to B(x) + d\xi(x)$, with $\xi(x)$ being a one-form gauge parameter. In this case, the dual field theory is a free scalar field theory.

The non-Abelian generalization of the theory of AST fields was found by Freedman and Townsend [5], and it is called the Freedman-Townsend (FT) model. In this theory, the AST field $B_{\mu\nu} = B^{i}{}_{\mu\nu}T_i$ transforms as the adjoint representation of the group G, where T_i are generators of G. It possesses the non-Abelian generalization of the AST gauge invariance [5, 6]. In this case, the dual scalar field theory is a nonlinear sigma model (NLSM) on the group manifold G, called the principal chiral model. NLSM in four dimensions have attracted interest as low-energy effective field theories of QCD. The NLSM on the coset spaces G/H naturally appear when the global symmetry is spontaneously broken down to its subgroup H [7]. The scalar-tensor duality can be extended to the duality between AST gauge theories coupled to Yang-Mills (YM) gauge fields and NLSM on coset spaces G/H [8].

Supersymmetry plays roles in realizing electric-magnetic duality and other types of duality at quantum level in gauge theories and string theories [1, 4]. In this paper, we investigate the supersymmetric extension of the scalar-tensor duality. The Abelian scalar-tensor duality in supersymmetric field theories was studied extensively [9, 10, 11]. The non-Abelian generalization of scalar-tensor duality in supersymmetric field theories was found in Ref. [12]. The dual field theories are the supersymmetric NLSM on the complex extension of the group manifold G, $G^{\mathbf{C}}$. This is related to the fact that the target spaces of supersymmetric NLSM must be Kähler [13]. In supersymmetric field theories, spontaneously broken global symmetry is usually realized by the complex extension of the coset spaces, but not G/H

itself [14]. We study the supersymmetric extension of the scalar-tensor duality for the complex coset spaces.

This paper is organized as follows. In §2, we recapitulate the non-Abelian scalar-tensor duality in bosonic theories. In §3, we recall the scalar-tensor duality in supersymmetric field theories in the case of the complex extension of the group manifold as the target space of NLSM [12]. The relation of the (quasi-)Nambu-Goldstone bosons in the NLSM and the boson fields in the AST gauge theory is clarified. In §4 we construct the supersymmetric extension of the duality of the non-Abelian AST field theory and the NLSM on a coset space based on the results in §2 and §3. §5 is devoted to discussion.

2 Non-Abelian scalar-tensor duality in bosonic theories

We begin by recapitulating the non-Abelian scalar-tensor duality [5] and its G/H generalization [8]. We consider non-Abelian AST fields $B_{\mu\nu}$ with the group G. Our notation of its Lie algebra $\mathcal{G}$ is

$$\begin{aligned} &T_i \in \mathcal{G}, \qquad i = 1, \cdots, \dim G, \\ &[T_i, T_j] = i f_{ij}{}^k T_k, \qquad \mathrm{tr}\,(T_i T_j) = c\delta_{ij}, \end{aligned} \tag{2.1}$$

where $f_{ij}{}^k$ are the structure constants, and c is a positive constant. We use the matrix notation for the AST field and the auxiliary vector field:

$$B_{\mu\nu}(x) \equiv B^i_{\mu\nu}(x) T_i, \qquad A_\mu(x) \equiv A^i_\mu(x) T_i, \tag{2.2}$$

whose mass dimensions are 1 and 2, respectively. The Lagrangian of the FT model in the first-order form is given by

$$\mathcal{L} = -\frac{1}{8c}\mathrm{tr}\left(\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma} + A_\mu A^\mu\right), \tag{2.3}$$

where $F_{\rho\sigma} \equiv \partial_\rho A_\sigma - \partial_\sigma A_\rho - i\lambda\,[A_\rho, A_\sigma]$, and λ is the coupling constant with dimension -1. This Lagrangian is invariant up to total derivative under the AST gauge transformation, defined by

$$\delta B_{\mu\nu} = D_\mu(A)\xi_\nu - D_\nu(A)\xi_\mu, \qquad \delta A_\mu = 0, \tag{2.4}$$

where $\xi_\mu(x) \equiv \xi^i_\mu(x)T_i$ is a vector field gauge parameter, and we have introduced the covariant derivative, $D_\mu(A)\xi_\nu = \partial_\mu\xi_\nu - i\lambda[A_\mu, \xi_\nu]$. In addition, the Lagrangian (2.3) has a *global* symmetry of the group G with the transformation

$$B_{\mu\nu} \to B'_{\mu\nu} = g^{-1}B_{\mu\nu}g, \qquad A_\mu \to A'_\mu = g^{-1}A_\mu g \tag{2.5}$$

with $g \in G$.

The second-order Lagrangian for $B^i_{\mu\nu}$ is obtained by eliminating A^i_μ (see Ref. [5]). We obtain the dual formulation of the AST gauge theory by eliminating $B^i_{\mu\nu}$ instead. It gives the flatness condition for A_μ, $F_{\mu\nu} = 0$. Its solution is given by

$$A_\mu = \frac{i}{\lambda}U^{-1}\partial_\mu U, \tag{2.6}$$

where $U(x) \in G$. Substituting (2.6) back into the Lagrangian (2.3), we get

$$\mathcal{L} = \frac{1}{8c\lambda^2}\mathrm{tr}\left(U^{-1}\partial_\mu U\right)^2. \tag{2.7}$$

This is the NLSM on the group manifold G, called the principal chiral model.

The generalization of the scalar-tensor duality to the case of a coset space G/H as the target space of the NLSM can be made by introducing the YM field for the subgroup H [8],

$$v_\mu \equiv v^a_\mu H_a \in \mathcal{H}. \tag{2.8}$$

Here we have decomposed the Lie algebra $\mathcal{G}$ into the subalgebra $\mathcal{H}$ and the coset generators as

$$T_i \in \mathcal{G}, \qquad H_a \in \mathcal{H}, \qquad X_I \in \mathcal{G} - \mathcal{H}, \qquad \mathrm{tr}\,(H_a X_I) = 0. \tag{2.9}$$

The AST fields and the auxiliary vector fields are defined in the same way as (2.2). The YM gauge transformations of the group H now read

$$\begin{aligned} v_\mu \quad &\to \quad v'_\mu = h^{-1}(x)\left(v_\mu + \frac{i}{e}\partial_\mu\right)h(x), \\ B_{\mu\nu} \to B'_{\mu\nu} &= h^{-1}(x)B_{\mu\nu}h(x), \qquad A_\mu \quad \to \quad A'_\mu = h^{-1}(x)A_\mu h(x), \end{aligned} \tag{2.10}$$

with $h(x) \in H$. Here e is the YM coupling constant with dimension zero. The first-order Lagrangian invariant under this gauge transformation is given by

$$\mathcal{L} = -\frac{1}{8c}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[B_{\mu\nu}F_{\rho\sigma}\left(A + \frac{e}{\lambda}v\right)\right] - \frac{1}{8c}\mathrm{tr}(A_\mu A^\mu), \tag{2.11}$$

where $F_{\mu\nu}\left(A + \frac{e}{\lambda}v\right) \equiv F_{\mu\nu} + \frac{e}{\lambda}\left(\partial_\mu v_\nu - \partial_\nu v_\mu - ie\left[v_\mu, v_\nu\right]\right) - ie\left(\left[A_\mu, v_\nu\right] + \left[v_\mu, A_\nu\right]\right)$.

The AST gauge transformation is modified from (2.4) after taking account of the coupling of $B^i_{\mu\nu}$ with the YM field v^a_μ. One just has to replace the covariant derivative $D_\mu(A)$ by $D_\mu(A + \frac{e}{\lambda}v)$:

$$\begin{aligned} &\delta B_{\mu\nu} = D_\mu\left(A + \frac{e}{\lambda}v\right)\xi_\nu - D_\nu\left(A + \frac{e}{\lambda}v\right)\xi_\mu, \\ &\delta A_\mu = 0, \qquad \delta v_\mu = 0. \end{aligned} \tag{2.12}$$

The case of H being G itself was considered in [5]. Their model is different from ours in the sense that it contains the kinetic term of the YM field.

Note that the existence of the gauge field v_μ in the Lagrangian (2.11) breaks explicitly the original G symmetry of the Lagrangian (2.3), since $v'_\mu = g^{-1}v_\mu g$ is not necessarily an element of $\mathcal{H}$.

Eliminating A_μ from the Lagrangian (2.11), we can get the second-order Lagrangian for $B_{\mu\nu}$ coupled to the YM field v_μ:

$$\mathcal{L} = -\frac{1}{8}\left(\tilde{G}^{\mu i}\tilde{K}^{ij}_{\mu\nu}\tilde{G}^{\nu j} + \frac{e}{\lambda}\epsilon^{\mu\nu\rho\sigma}B^i_{\mu\nu}v^i_{\rho\sigma}\right), \tag{2.13}$$

where we have defined the field strength $\tilde{G}^{\mu i} \equiv \epsilon^{\mu\nu\rho\sigma}D_\nu B^i_{\rho\sigma} \equiv \epsilon^{\mu\nu\rho\sigma}(\partial_\nu B^i_{\rho\sigma} + ef^{ijk}v^j_\nu B^k_{\rho\sigma})$, and $v^i_{\mu\nu}$ is the YM field strength. Here, $\tilde{K}^{ij}_{\mu\nu}$ is related to $K^{\mu\nu ij} \equiv g^{\mu\nu}\delta^{ij} - \lambda f^{ijk}\epsilon^{\mu\nu\rho\sigma}B^k_{\rho\sigma}$ through $\tilde{K}^{ik}_{\mu\rho}K^{\rho\nu kj} = \delta^\nu_\mu\delta^{ij}$.

We now proceed to the derivation of the dual scalar field theory equivalent to the AST field theory (2.13). Elimination of $B^i_{\mu\nu}$ in the Lagrangian (2.11) yields the constraint

$$F_{\mu\nu}\left(A + \frac{e}{\lambda}v\right) = 0. \tag{2.14}$$

Its solution is

$$A_\mu + \frac{e}{\lambda}v_\mu = \frac{i}{\lambda}U(x)^{-1}\partial_\mu U(x), \quad U(x) \in G. \tag{2.15}$$

Substituting this expression back into Eq. (2.11), we get

$$\mathcal{L} = \frac{1}{8c\lambda^2}\mathrm{tr}(U^{-1}D_\mu U)^2. \tag{2.16}$$

where we have introduced the covariant derivative, $D_\mu U \equiv \partial_\mu U + ieUv_\mu$. The Lagrangian (2.16) has the *global* $G_L\times$ *local* H_R symmetry [the subscript L (R) means left (right) action], given by

$$U(x) \to gU(x), \quad v_\mu(x) \to v_\mu(x), \tag{2.17}$$

with $g \in G$, and

$$U(x) \to U(x)h(x), \quad v_\mu \to h^{-1}(x)\left(v_\mu + \frac{i}{e}\partial_\mu\right)h(x), \tag{2.18}$$

with $h(x) \in H$, respectively. The global G transformation (2.17) is hidden in the AST gauge theory. Eliminating the auxiliary field v_μ by using the equation of motion, we obtain the usual form of the NLSM on G/H [7, 15].

There is an alternative way of writing the first-order Lagrangian. We replace the auxiliary field A_μ as

$$A_\mu \to A_\mu - \frac{e}{\lambda}v_\mu, \tag{2.19}$$

by field redefinition. The Lagrangian (2.11) becomes

$$\mathcal{L} = -\frac{1}{8c}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left(B_{\mu\nu}F_{\rho\sigma}\right) - \frac{1}{8c}\mathrm{tr}\left[\left(A_\mu - \frac{e}{\lambda}v_\mu\right)\left(A^\mu - \frac{e}{\lambda}v^\mu\right)\right]. \tag{2.20}$$

This expression of the Lagrangian was obtained in [8]. Note that it is invariant under the AST gauge transformation (2.4) with $\delta v_\mu = 0$, instead of (2.12). The YM gauge transformation of A_μ takes the usual form: $A_\mu \to A'_\mu = h^{-1}(x)\left(A_\mu + \frac{i}{\lambda}\partial_\mu\right)h(x)$.

3 Supersymmetric extension of non-Abelian scalar-tensor duality for group manifolds

After we recapitulate the non-Abelian scalar-tensor duality for the group manifold in supersymmetric field theories [12], we discuss the correspondence of fields in both theories. In supersymmetric field theories, the AST fields $B_{\mu\nu}(x)$ belong to the (anti-)chiral spinor superfield $B_\alpha(x,\theta,\bar\theta)$ $[\bar B_{\dot\alpha}(x,\theta,\bar\theta)]$, satisfying the supersymmetric constraints [9]

$$\bar D_{\dot\alpha} B_\beta(x,\theta,\bar\theta)=0, \qquad D_\alpha \bar B_{\dot\beta}(x,\theta,\bar\theta)=0. \tag{3.1}$$

They can be expanded in terms of component fields as

$$\begin{aligned} B^\alpha(y,\theta) &= \psi^\alpha(y)+\frac{1}{2}\theta^\alpha(C(y)+iD(y))+\frac{1}{2}(\sigma^{\mu\nu})^{\alpha\beta}\theta_\beta B_{\mu\nu}(y)+\theta\theta\lambda^\alpha(y),\\ \bar B_{\dot\alpha}(y^\dagger,\bar\theta) &= \bar\psi_{\dot\alpha}(y^\dagger)+\frac{1}{2}\bar\theta_{\dot\alpha}(C(y^\dagger)-iD(y^\dagger))+\frac{1}{2}(\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta}B_{\mu\nu}(y^\dagger)+\bar\theta\bar\theta\bar\lambda_{\dot\alpha}(y^\dagger), \end{aligned} \tag{3.2}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$ and $\bar D_{\dot\alpha}=\partial/\partial\bar\theta^{\dot\alpha}$ (or $y^{\mu\dagger}=x^\mu - i\theta\sigma^\mu\bar\theta$ and $D_\alpha=-\partial/\partial\theta^\alpha$), and $(\sigma^{\mu\nu})^\alpha{}_\beta=\frac{1}{4}(\sigma^\mu\bar\sigma^\nu-\sigma^\nu\bar\sigma^\mu)^\alpha{}_\beta$ (see [16]). The mass dimension of B_α is $\frac{1}{2}$.[1]

We consider non-Abelian AST fields $B_{\mu\nu}$ with the group G [see (2.1)]. $B_{\mu\nu}$ belongs to $\mathcal{G}$-valued (anti-)chiral spinor superfields, $B_\alpha(x,\theta,\bar\theta)=B^i_\alpha(x,\theta,\bar\theta)T_i$ $[\bar B_{\dot\alpha}(x,\theta,\bar\theta)=\bar B^i_{\dot\alpha}(x,\theta,\bar\theta)T_i]$.

To construct the supersymmetric extension of the FT model, we introduce a $\mathcal{G}$-valued auxiliary vector superfield $A(x,\theta,\bar\theta)=A^i(x,\theta,\bar\theta)T_i$, satisfying the constraint $A^{i\dagger}=A^i$. Its field strengths are (anti-)chiral spinor superfields,

$$W_\alpha = -\frac{1}{4\lambda}\bar D\bar D(e^{-\lambda A}D_\alpha e^{\lambda A}), \qquad \bar W_{\dot\alpha}=\frac{1}{4\lambda}DD(e^{\lambda A}\bar D_{\dot\alpha}e^{-\lambda A}). \tag{3.3}$$

The first-order Lagrangian can be written as [10, 12]

$$\mathcal{L}=-\frac{1}{2c}\left[\int d^2\theta\ \mathrm{tr}\,(W^\alpha B_\alpha)+\int d^2\bar\theta\ \mathrm{tr}\,(\bar W_{\dot\alpha}\bar B^{\dot\alpha})\right]+\frac{1}{4c}\int d^4\theta\ \mathrm{tr}\,A^2. \tag{3.4}$$

This Lagrangian is justified by constructing the supersymmetric AST gauge transformation which leaves it invariant. To this end we define covariant spinor derivative $\mathcal{D}_\alpha = D_\alpha + [e^{-\lambda A}D_\alpha e^{\lambda A},\ \cdot\ ]$. The AST gauge transformation is parameterized by a $\mathcal{G}$-valued vector superfield $\Omega(x,\theta,\bar\theta)=\Omega^i(x,\theta,\bar\theta)T_i$, satisfying the constraint $\Omega^{i\dagger}=\Omega^i$,

$$\begin{aligned} &\delta B_\alpha = -\frac{i}{4}\bar D\bar D\mathcal{D}_\alpha(e^{-\lambda A}\Omega), \qquad \delta\bar B_{\dot\alpha}=-\frac{i}{4}DD\bar{\mathcal{D}}_{\dot\alpha}(\Omega e^{-\lambda A}),\\ &\delta A = 0. \end{aligned} \tag{3.5}$$

[1] We summarize the dimensions of (super)fields introduced in this and the following sections: $[B_\alpha]=\frac{1}{2}$, $[B_{\mu\nu}]=1$, $[A]=1$, $[A_\mu]=2$, $[W_\alpha]=\frac{5}{2}$, $[V]=0$ and $[v_\mu]=1$. Here, V is YM vector superfields introduced in the next section. The dimensions of coupling constants are $[\lambda]=-1$ and $[e]=0$ as in the bosonic case.

This transformation is Abelian though Ω is $\mathcal{G}$-valued. The Lagrangian (3.4) is invariant under a *global* G-transformation

$$\begin{aligned} &B_\alpha \to B'_\alpha = g^{-1} B_\alpha g, \qquad \bar{B}_{\dot\alpha} \to \bar{B}'_{\dot\alpha} = g^{-1} \bar{B}_{\dot\alpha} g, \\ &A \to A' = g^{-1} A g, \qquad W_\alpha \to W'_\alpha = g^{-1} W_\alpha g, \end{aligned} \tag{3.6}$$

with $g \in G$.

The equation of motion for the auxiliary superfields A takes a complicated form [10]

$$\mathcal{D}^\alpha B_\alpha + e^{-\lambda A} \bar{\mathcal{D}}_{\dot\alpha} \bar{B}^{\dot\alpha} e^{\lambda A} = -A, \tag{3.7}$$

where $\mathcal{D}_\alpha B_\beta \equiv D_\alpha B_\beta + \{e^{-\lambda A} D_\alpha e^{\lambda A}, B_\beta\}$. If we eliminate A by solving Eq. (3.7), we obtain the second-order Lagrangian for B_α, which we call the supersymmetric theory of the FT model. In practice it is difficult to solve Eq. (3.7) explicitly, and hence we do not write the Lagrangian for B_α explicitly.

On the other hand, if we eliminate $B_\alpha(x,\theta,\bar\theta)$, we obtain the dual NLSM as follows. The equation of motion of $B_\alpha(x,\theta,\bar\theta)$ reads

$$-4\lambda W_\alpha(x,\theta,\bar\theta) = \bar{D}\bar{D}(e^{-\lambda A} D_\alpha e^{\lambda A}) = 0, \tag{3.8}$$

expressing that A is a pure gauge. The solution is written as

$$e^{\lambda A(x,\theta,\bar\theta)} = e^{\phi^\dagger(x,\theta,\bar\theta)} e^{\phi(x,\theta,\bar\theta)}, \qquad \bar{D}_{\dot\alpha} \phi(x,\theta,\bar\theta) = 0. \tag{3.9}$$

Here $\phi = \phi^i T_i$ is a $\mathcal{G}$-valued chiral superfield. Taking account of Eq. (3.8) and substituting (3.9) back into the Lagrangian (3.4), we obtain the Lagrangian for ϕ [12]

$$\mathcal{L} = \int d^4\theta \; K(\phi, \phi^\dagger) = \int d^4\theta \; \frac{1}{4c\lambda^2} \mathrm{tr} \, \log^2 (e^{\phi^\dagger} e^{\phi}). \tag{3.10}$$

We have thus obtained the NLSM which is dual to the supersymmetric theory of the FT model.

In the Lagrangian (3.10), K is the Kähler potential of the target space of the NLSM. K can be expanded in powers of ϕ^i and $\phi^{i\dagger}$. We have

$$\begin{aligned} \mathcal{L} = \int d^4\theta \frac{1}{2\lambda^2} \Big[\phi^{i\dagger}\phi^i - \frac{1}{24} f_{ml}{}^i f_{jk}{}^l (&\phi^{m\dagger}\phi^{j\dagger}\phi^k\phi^i \\ &+ 2\phi^{i\dagger}\phi^{m\dagger}\phi^{j\dagger}\phi^k + 2\phi^{k\dagger}\phi^i\phi^j\phi^m) + \cdots \Big] . \end{aligned} \tag{3.11}$$

One finds that the dual field theory is the free Wess-Zumino model when the group G is Abelian.

Since the scalar components of ϕ^i are complex, the target space of this NLSM is $G^{\mathbf{C}}$, the complex extension of G. The Lagrangian (3.10) is invariant under the global action of a group $G \times G$

$$e^{\phi} \to e^{\phi'} = g_L e^{\phi} g_R, \qquad (g_{\rm L}, g_{\rm R}) \in G \times G. \tag{3.12}$$

Note that the Lagrangian (3.10) is not invariant under $G^{\mathbf{C}} \times G^{\mathbf{C}}$, though the target space is $G^{\mathbf{C}}$. We find from Eqs. (3.6) and (3.9) that the right action of Eq. (3.12) arises from the original global symmetry G of the AST gauge theory; on the other hand, the left action of Eq. (3.12) is a *hidden* global symmetry, preserving Eq. (3.9).

The Lagrangian (3.10) allows a simple interpretation from the view-point of the supersymmetric nonlinear realization of a global symmetry. In supersymmetric field theories, spontaneous symmetry breaking is caused by the superpotential. Since the superpotential is a holomorphic function of chiral superfields, its symmetry G is promoted to $G^{\mathbf{C}}$. Correspondingly, there appear quasi-Nambu-Goldstone (QNG) bosons in addition to ordinary Nambu-Goldstone (NG) bosons [17]. The low-energy effective Lagrangian of these massless bosons and their fermionic superpartners is a supersymmetric NLSM whose target manifold is a Kähler manifold [13]. It is non-compact due to the existence of QNG bosons [18]. Since the target space of the Lagrangian (3.10) is $G^{\mathbf{C}}$, there are the same numbers of NG and QNG bosons, for completely broken G.

We examine the relation of the fields in the AST gauge theory with the NG and QNG bosons in the dual NLSM. The relation can be worked out explicitly in the Abelian case as follows. The AST gauge transformation of $B_{\mu\nu}$ is given by $\delta B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$, where ξ_μ is the vector component of the parameter vector superfield Ω. Using the AST gauge transformation of the other fields, we can take the Wess-Zumino gauge, $\psi_\alpha = D = 0$. The second-order action for B_α, obtained by eliminating A in the Lagrangian (3.4) for the Abelian case, is given by [9]

$$S = -\int d^4x \int d^4\theta \, \frac{1}{2} G^2 = \int d^4x \left(-\frac{1}{2} \partial_\mu C \partial^\mu C - i \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{4} G^\mu G_\mu \right). \tag{3.13}$$

Here $G(x,\theta,\bar{\theta})$ is the field strength of B_α defined by $G \equiv \frac{1}{2}(D^\alpha B_\alpha + \bar{D}_{\dot{\alpha}}\bar{B}^{\dot{\alpha}}) = \theta\sigma^\mu\bar{\theta}G_\mu + \cdots$, where $G^\mu \equiv \varepsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma}$. We find from this action that $B_{\mu\nu}$ corresponds to the NG boson and C to the QNG boson. For the non-Abelian case, the second-order Lagrangian is an interacting field theory, and the relation between NG (QNG) bosons and $B_{\mu\nu}$ (C) is less straightforward.

4 Supersymmetric extension of non-Abelian scalar-tensor duality for coset spaces

4.1 Anti-symmetric tensor gauge theory coupled to Yang-Mills fields

We generalize the scalar-tensor duality to the case of coset space. In supersymmetric theories, we introduce YM vector superfields $V(x,\theta,\bar{\theta}) = V^a(x,\theta,\bar{\theta})H_a$, where H_a are generators of a *subgroup* H of G. We decompose the Lie algebra as Eq. (2.9).

As mentioned previously, there are two expressions for generalizing the FT model to the case of coset space. We take the second expression (2.20). Then, we do not need to modify the AST gauge transformation and the field strength.

As the YM gauge transformation of A_μ in the bosonic case, A transforms like the YM gauge field V, replacing eV by λA. The YM gauge transformation is

$$\begin{aligned} &B_\alpha \to B'_\alpha = e^{-i\Lambda}B_\alpha e^{i\Lambda}, && \bar{B}_{\dot{\alpha}} \to \bar{B}'_{\dot{\alpha}} = e^{-i\Lambda^\dagger}\bar{B}_{\dot{\alpha}}e^{i\Lambda^\dagger}, \\ &e^{eV} \to e^{eV'} = e^{-i\Lambda^\dagger}e^{eV}e^{i\Lambda}, && e^{\lambda A} \to e^{\lambda A'} = e^{-i\Lambda^\dagger}e^{\lambda A}e^{i\Lambda}, \end{aligned} \tag{4.1}$$

where the gauge parameter is a $\mathcal{H}$-valued chiral superfield $\Lambda(x,\theta,\bar{\theta}) = \Lambda^a(x,\theta,\bar{\theta})H_a$, satisfying $\bar{D}_{\dot{\alpha}}\Lambda(x,\theta,\bar{\theta}) = 0$. The field strengths of A are the same as (3.3). They transform as

$$W_\alpha \to W'_\alpha = e^{-i\Lambda}W_\alpha e^{i\Lambda}, \qquad \bar{W}_{\dot{\alpha}} \to \bar{W}'_{\dot{\alpha}} = e^{-i\Lambda^\dagger}\bar{W}_{\dot{\alpha}}e^{i\Lambda^\dagger}, \tag{4.2}$$

under the YM gauge transformation (4.1).

We have already given the Lagrangian in the first-order form, Eq. (3.4), in the case in which there is no YM gauge field. A generalization of the Lagrangian, in which G is fully gauged by supersymmetric YM fields, is given in Ref. [12]. The dual field theory is a massive YM field theory.

We construct the model in which the subgroup H of G is gauged by YM fields. The answer is

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2c}\left[\int d^2\theta \ \mathrm{tr}\,(W^\alpha B_\alpha) + \int d^2\bar{\theta} \ \mathrm{tr}\,(\bar{W}_{\dot{\alpha}}\bar{B}^{\dot{\alpha}})\right] \\ &\quad + \frac{1}{4c\lambda^2}\int d^4\theta \ \mathrm{tr}\ \log^2(e^{-eV}e^{\lambda A}).\end{aligned} \tag{4.3}$$

If we put $V = 0$, we recover the Lagrangian (3.4). We should note that the global action of G, Eq. (3.6), is explicitly *broken* by the partial gauging of the subgroup H of G; the $\mathcal{H}$-valuedness of V is not kept under the G-action $V \to V' = g^{-1}Vg$.

The equation of motion of the auxiliary field A is difficult to solve, and we do not write the second-order Lagrangian for B_α.

Here we discuss the field redefinition corresponding to (2.19) in the bosonic case. In the second alternative corresponding to (2.20), the AST gauge transformation and the field strengths are not modified. The Lagrangian in the first alternative corresponding to (2.11) is obtained from the Lagrangian (4.3) by the field redefinition of A,

$$e^{\lambda A} \to e^{eV}e^{\lambda A}. \tag{4.4}$$

Using the Hausdorff formula, this can be rewritten as

$$A \to A + \frac{e}{\lambda}V + \frac{e}{2}[A, V] + \cdots. \tag{4.5}$$

The Lagrangian is

$$\mathcal{L} = -\frac{1}{2c}\left[\int d^2\theta \ \mathrm{tr}\,(\tilde{W}^\alpha B_\alpha) + \int d^2\bar{\theta} \ \mathrm{tr}\,(\tilde{\bar{W}}_{\dot{\alpha}}\bar{B}^{\dot{\alpha}})\right] + \frac{1}{4c}\int d^4\theta \ \mathrm{tr}\, A^2. \tag{4.6}$$

Here $\tilde{W}^\alpha$ and $\tilde{\bar{W}}_{\dot{\alpha}}$ are defined by

$$\begin{aligned}\tilde{W}_\alpha &= -\frac{1}{4\lambda}\bar{D}\bar{D}\left[e^{-\lambda A}e^{-eV}D_\alpha(e^{eV}e^{\lambda A})\right], \\ \tilde{\bar{W}}_{\dot{\alpha}} &= \frac{1}{4\lambda}DD\left[e^{eV}e^{\lambda A}\bar{D}_{\dot{\alpha}}(e^{-\lambda A}e^{-eV})\right],\end{aligned} \tag{4.7}$$

which are obtained from (3.3) using (4.4). The AST gauge transformation (3.5) is also modified by (4.4) to

$$\delta B_\alpha = -\frac{i}{4}\bar{D}\bar{D}\tilde{\mathcal{D}}_\alpha(e^{-\lambda A}e^{-eV}\Omega), \qquad \delta \bar{B}_{\dot{\alpha}} = -\frac{i}{4}DD\tilde{\bar{\mathcal{D}}}_{\dot{\alpha}}(\Omega e^{-\lambda A}e^{-eV}),$$
$$\delta A = 0, \qquad \delta V = 0, \tag{4.8}$$

with $\tilde{\mathcal{D}}_\alpha = D_\alpha + [e^{-\lambda A}e^{-eV}D_\alpha(e^{eV}e^{\lambda A}), \cdot]$. The YM gauge transformation of A becomes

$$A \to A' = e^{-i\Lambda}Ae^{i\Lambda}. \tag{4.9}$$

At first sight, one might think that this is inconsistent with the reality condition, but it is not the case; $e^{-eV}e^{\lambda A}$ and V satisfy the reality condition under the redefinition (4.4).

4.2 Dual nonlinear sigma model on complex coset spaces

To obtain the dual NLSM, we return to the Lagrangian (4.3). Elimination of $B_\alpha(x,\theta,\bar{\theta})$ by its equation of motion again gives

$$e^{\lambda A(x,\theta,\bar{\theta})} = e^{\phi^\dagger(x,\theta,\bar{\theta})}e^{\phi(x,\theta,\bar{\theta})}, \qquad \bar{D}_{\dot{\alpha}}\phi(x,\theta,\bar{\theta}) = 0, \tag{4.10}$$

where ϕ is a $\mathcal{G}$-valued chiral superfield. Proceeding in the same way as in §3, we obtain the NLSM whose Kähler potential is

$$K(\phi,\phi^\dagger,V) = \frac{1}{4c\lambda^2}\mathrm{tr}\,\log^2(e^{-eV}e^{\phi^\dagger}e^{\phi}). \tag{4.11}$$

As in the bosonic case, the NLSM (4.11) is invariant under the global transformation of G

$$e^\phi \to e^{\phi'} = ge^\phi, \qquad g \in G \tag{4.12}$$

and the gauge transformation of H

$$e^{eV} \to e^{eV'} = h^\dagger e^{eV}h, \qquad e^\phi \to e^{\phi'} = e^\phi h,$$
$$h = e^{i\Lambda^a(x,\theta,\bar{\theta})H_a}, \qquad \bar{D}_{\dot{\alpha}}\Lambda^a(x,\theta,\bar{\theta}) = 0. \tag{4.13}$$

The global action (4.12) of G corresponds to the left action in Eq. (3.12), which does not correspond to any symmetry in the AST gauge theory. On the other hand, the local action (4.13) of H corresponds to the right action in Eq. (3.12) and its origin is, of course, the gauge transformation (4.1). (4.11) can be interpreted as the Lagrangian of the dual NLSM formulated in terms of the *hidden local symmetry* [15].

The physical degrees of freedom can be found as follows. Decompose $\phi(x,\theta,\bar{\theta})$ into the $\mathcal{H}$-valued part and the rest as

$$\begin{aligned} &e^{\phi(x,\theta,\bar{\theta})} = \xi(x,\theta,\bar{\theta})h(x,\theta,\bar{\theta}), \\ &\xi = e^{i\varphi^I(x,\theta,\bar{\theta})X_I}, \qquad h = e^{i\alpha^a(x,\theta,\bar{\theta})H_a}, \end{aligned} \tag{4.14}$$

where X_I are the coset generators, and $\varphi^I(x,\theta,\bar{\theta})$ and $\alpha^a(x,\theta,\bar{\theta})$ are chiral superfields. By fixing the gauge as

$$e^{eV} = h^\dagger e^{eV_0} h, \qquad h = e^{i\alpha^a(x,\theta,\bar{\theta})H_a}, \tag{4.15}$$

we obtain the Kähler potential

$$K(\phi,\phi^\dagger,V_0) = \frac{1}{4c\lambda^2}\mathrm{tr}\ \log^2(e^{-eV_0}\xi^\dagger\xi). \tag{4.16}$$

This Lagrangian is invariant under the global G-transformation, defined by

$$\begin{aligned} &\xi \to \xi' = g\xi h(g,\xi), \\ &e^{eV_0} \to e^{eV_0{}'} = h^\dagger(g,\xi)e^{eV_0}h(g,\xi), \qquad g \in G, \end{aligned} \tag{4.17}$$

where the H-valued chiral superfield $h(g,\xi(x,\theta,\bar{\theta}))$ is a compensator needed for recovering the decomposition in Eq. (4.14).

ξ is a coset representative of $G^{\mathbf{C}}/H^{\mathbf{C}}$ and the bosonic parts of $\varphi^I(x,\theta,\bar{\theta})$ parameterize $G^{\mathbf{C}}/H^{\mathbf{C}}$. $\alpha^a(x,\theta,\bar{\theta})$ in Eq. (4.14) has been absorbed into V^a as a result of the supersymmetric Higgs mechanism. Each chiral superfield φ^I in this NLSM contains the same numbers of the NG and QNG bosons. The relation to the general theory of the supersymmetric nonlinear realizations [14] is, however, obscure at this stage, since we are unable to eliminate V explicitly.

5 Discussion

We have constructed the supersymmetric extension of the duality between AST gauge theories and NLSM on coset spaces. The both models have apparently different manifest symmetries.

It is useful to clarify the interplay of the symmetries of the both models, which we do in the bosonic case; the situation is the same in the supersymmetric case. The relation of symmetries of the AST gauge theories and the NLSM is summarized in Table 1 for both the FT model and the H_R-gauged FT model.

model	FT model	$\longleftrightarrow$	NLSM on G		
symmetry	*global* $G_R \times$ *AST gauge*		*global* $G_L \times$ *global* G_R		
model	H_R-gauged FT model	$\longleftrightarrow$	H_R-gauged NLSM on G	$\longleftrightarrow$	G/H NLSM
symmetry	*local* $H_R \times$ *AST gauge*		*global* $G_L \times$ *local* H_R		*global* G_L

Table 1: Relation between the symmetries of the AST gauge theories and those of the NLSM. The upper table shows the FT scalar-tensor duality. The G/H scalar-tensor duality in the lower table is obtained by gauging the FT model.

It deserves making comments on a few points:

i) The AST gauge symmetry is hidden in the gauged NLSM.

ii) The global symmetry G_R, Eq. (2.5) [(3.6) in the SUSY case], of the FT model is broken in the gauged FT model.

iii) The global symmetry G_L, Eq. (2.17)[(4.12)] of gauged NLSM is hidden in the gauged FT model.

iv) On the other hand, the local symmetry H_R, Eq. (2.18)[(4.13)], of the gauged NLSM is hidden in the NLSM on G/H.

i) and iii) are also true in the non-gauged models.

Existence of solitons plays a role in the duality in supersymmetric gauge theories [1]. Perhaps solitons also play a role in the scalar-tensor duality. From this point of view it is of interest to extend the scalar-tensor duality by including higher derivative terms in the NLSM and supersymmetric NLSM [19].

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